# **Body Segment Parameters**<sup>1</sup>

# A Survey of Measurement Techniques

Human motor activity is determined by the response of the subject to constantly changing external and internal stimuli. The motor response has a definite pattern which can be analyzed on the basis of temporal, kinematic and kinetic factors.

Temporal factors are those related to time: cadence (tempo) or the number of movements per unit time (minute or second), the variability of successive durations of motion, and temporal pattern. The temporal pattern of each movement consists of two or more phases. The relative duration of these phases and their interrelationships are indicative characteristics of the movement under consideration. For example, in walking, two basic time phases may be noted, the stance phase when the leg is in contact with the ground and the swing phase. The ratio of swing-phase time to stance-phase time is one of the basic characteristics of gait.

The kinematic analysis of movement can be accomplished by studying the linear and angular displacements of the entire body, the joints (neck, shoulder, elbow, wrist, hip, knee, ankle) and the segments (head, upper arm, forearm, hand, thigh, shank, foot). For the purpose of

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investigation, the most important kinematic characteristics are: the paths of motion, linear and angular displacement curves, amplitudes or ranges of motion, the instantaneous and average velocities and their directions, and finally the linear and angular accelerations of the body segments under investigation. Information on these criteria can be obtained readily from objective (optical or electrical) recordings of the movements of a subject.

The kinetic analysis is concerned with the influence of different forces and moments acting on the body or a body segment during the performance of a given activity. To determine these forces and moments, accurate data on the mass (weight), location of mass centers (centers of gravity), and the mass moments of inertia of the subject's body segments are required.

At present there are limited data on body segment parameters, especially those for American subjects. Such data available are based on studies made on a limited number of dissected male cadavers. This cannot be regarded as a representative sample for our normal population with its wide range of age and difference of body build. There are no data available on female subjects in the United States.

A precise knowledge of these body segment parameters has many applications, such as in the design of work activities or the improvement of athletic performances. It has particular value in understanding orthopedic and prosthetic problems. It would result in a better design of braces and prosthetic devices and more reliable methods for their adjustment. From these data it would also be possible to develop more precise and effective procedures for the evaluation of braces and artificial limbs. These procedures would replace the use of subjective ratings on performance by an amputee or a disabled person.

The information on body segment parameters obtained by simple clinical methods can be very useful in general medical practice. It would provide a tool for the determination of:

1. body segment growth and decay in normal and abnormal conditions;

2. body segment density changes in normal and pathological cases;

3. body mass distribution asymmetry;

4. more precise body composition (fat, bones, muscles).

The aim of this article is to give a brief review of the methods used by different investigators for the determination of body segment parameters. Since some of the first treatises and papers are no longer available, we include some tables and figures which summarize the data obtained by some of the earlier researchers.

# EARLY EFFORTS

Since ancient times there has existed an intense curiosity about the mass distribution of the human body and the relative proportions of its various segments. Those professions which had to select or classify subjects of varying body build were particularly interested in the problem. In spite of individual differences between particular subjects there are many characteristics which are common to all normal human beings. Thus the lower extremities are longer and heavier than the upper extremities, the upper arm is larger than the forearm, the thigh is larger than the shank, and other similar relationships.

Historically this interest was first directed to the length relationships between the body segments. To characterize these relationships certain rules and canons were promulgated. Each canon has its own standard unit of measure or module. Sometimes the dimension of a body segment or component parts of a body segment were used as modules and occasionally the module was based on some abstract deduction.

The oldest known module is the distance measured between the floor (sole) and the ankle joint. This module was used in Egypt some time around the period 3000 B.C. On this

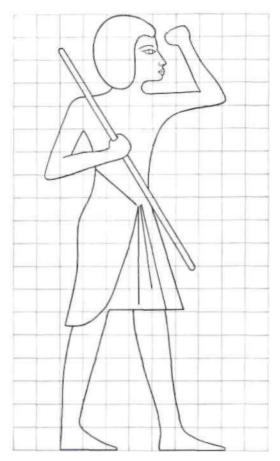


Fig. 1. Egyptian middle finger canon.

basis, the height of the human figure was set equal to 21.25 units. Several centuries later in Egypt a new module, the length of the middle finger, was introduced. In this instance body height was set equal to 19 units. This standard was in use up until the time of Cleopatra.

In the fifth century B.C., Polyclitus, a Greek sculptor, introduced as a module the width of the palm at the base of the fingers. He established the height of the body from the sole of the foot to the top of the head as 20 units, and on this basis the face was 1/10 of the total body height, the head 1/8, and the head and neck together 1/6 of the total body height. In the first century B.C., Vitruvius, a Roman architect, in his research on body proportions found that body height was equal to the arm spread—the distance between the tips of the middle fingers

with arms outstretched. The horizontal lines tangent to the apex of the head and the sole of the foot and the two vertical lines at the finger tips formed the "square of the ancients." This square was adopted by Leonardo da Vinci. He later modified the square by changing the position of the extremities and scribing a circle around the human figure.

Diirer (1470-1528) and Zeising (1810-1876) based their canons on mathematical abstracts which were not in accordance with any actual relationships.

At the beginning of the twentieth century, Kollmann tried to introduce a decimal stand-

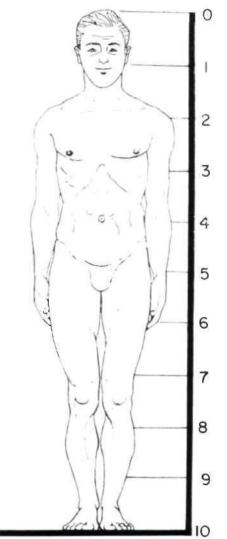


Fig. 2. Kollmann's decimal canon.

ard by dividing the body height into ten equal parts. Each of these in turn could be subdivided into ten subunits. According to this standard, the head height is equal to 13 of these smaller units: seated height, 52-53; leg length, 47; and the whole arm, 44 units.

# PREVIOUS STUDIES IN BODY PARAMETERS

Starting with the early investigators, the idea has prevailed that volumetric methods are best for determining relationships between body segments. There were basically two methods which were used for the determination of the volume of the body segments: (1) body segment immersion, and (2) segment zone measurement or component method. In these methods it is assumed that the density or specific gravity of any one body segment is homogeneous along its length. Hence the mass of the segment can be found by multiplying its volume by its density.

# IMMERSION METHOD

Harless in Germany first used the immersion method. In 1858 he published a text book on *Plastic Anatomy*, and in 1860 a treatise, *The Static Moments of the Human Body Limbs*. In his investigations, Harless dissected five male cadavers and three female cadavers. For his final report, however, he used only the data gathered on two of the subjects.

The immersion method involves determining how much water is displaced by the submerged segment. Previous researchers, including Harless, have relied on the measurement of the overflow of a water tank to find the volume of water displaced.

Harless started his studies with the determination of the absolute and relative lengths of the body and its segments. The absolute lengths were measured in centimeters. For determining the relative lengths, Harless used the hand as a standard unit. The standard hand measurement was equal to the distance from the wrist joint to the tip of the middle finger of the right hand. Later Harless also used the total height of the body as a relative unit of length. In the more recent studies on body parameters, this unit is accepted as the basis for the proportions of the various segment lengths. The results of Harless' studies are shown in Table 1. For obtaining the absolute weights of the body segments, Harless used the gram as the standard. As a unit for relative weights, he first decided to use the weight of the right hand, but later established as his unit the one thousandth part of the total body weight. His results are given in Table 2.

In a very careful way Harless determined the volume and density (specific gravity) of the body segments. The results of these measurements are presented in Table 3.

To determine the location of mass centers (centers of gravity), Harless used a wellbalanced board on which the segment was moved until it was in balance. The line coincident with the fulcrum axis of the board was marked on the segment and its distance from proximal and distal joints determined. The location of the mass center was then expressed as a ratio assuming the segment length to be equal to one. Harless also tried to determine the location of segment mass center from the apex of the head by assuming that the body height is equal to 1,000. The data for one subject are shown in Table 4. From the table, the asymmetry of the subject becomes evident.

To visualize the mass distribution of the human body, Harless constructed the model shown in Figure 3. The linear dimensions of the links of the model are proportional to the segment lengths; the volumes of the spheres are proportional to segment masses. The centers of

	Absolute Te	nothe in con	Relative Lengths				
	Absolute Le	Absolute Lengths in cm.		gth = 1,000	Body Height = 1,000		
	G	К	G	к	G	ĸ	
Head and neck	21.2	20.2	1.275	1.08	122.7	120.0	
Upper trunk	41.0	40.0	1.900	2.14	225.8	238.5	
Lower trunk	13.5	17.5	0,690	0.94	81.1	104.5	
Upper arm	36.4	30.3	1.792	1.62	211.1	180.7	
Forearm	29.9	26.2	1.471	1.40	173.1	156.5	
Hand	20.3	18.7	1.000	1.00	117.6	111.7	
Thigh	44.9	42.3	2.210	2.26	260.0	252.0	
Shank	42.9	38.1	2.111	2.03	248.4	227.0	
Foot	6.0	9.7	0.295	0.52	34.7	58.0	
Whole body	172.68	167.7	8.500	8.97	1,000	1,000	

TABLE 1. ABSOLUTE AND RELATIVE LENGTHS OF BODY SEGMENTS (AFTER HARLESS)

TABLE 2. ABSOLUTE AND RELATIVE WEIGHTS OF BODY SEGMENTS (AFTER HARLESS)

	Abashida Wa		Relative Weights					
	Absolute Weights in grams		Hand	= 1,000	Body Weight = 1,000.00			
	Cadaver 1	Cadaver 2	Cadaver 1	Cadaver 2	Cadaver 1	Cadaver 2		
Head	4,555	3,747	8.435	9.529	71.20	75.11		
Upper trunk	23,055	17,779	42.694	45.209	360.40	356.43		
Lower trunk	6,553	4,868	12.145	12.380	102.43	97.56		
Upper arm	2,070	1,448	3.833	3.682	32.35	29.04		
Forearm	1,160	795.5	2.148	2.023	18.13	15.94		
Hand	540	383.6	1.000	1.000	8.44	7.69		
Thigh	7,165	5,887	13.252	14.972	112.00	118,00		
Shank	2,800	2,247.5	5.185	5.716	43.77	45.04		
Foot	1,170	985.2	2.167	2.505	18.28	19.74		
Both upper extremities	7,540	5,254			117.86	105.34		
Both lower extremities	22,270	18,239.4			348.11	365.56		
Whole body	63,970	49,895	118.4	126.9	1,000.00	1,000.00		

Segment	Male (3 Cadavers)	Female (2 Cadavers)	Mean
Upper arm	1.0880	1.0596	1.0766
Forearm	1.1086	1.0714	1.0937
Hand	1.1126	1.1130	1.1128
Thigh	1.0686	1.0541	1.0628
Shank	1.1002	1.0822	1.0930
Foot	1.0893	1.1006	1.0922
Head	1.0851*	1.1300*	1.1107

TABLE 3. DENSITY OF THE BODY SEGMENTS (IN GR PER CM<sup>3</sup>) (AFTER HARLESS)

\* Data on one subject only.

TABLE 4. LOCATION OF THE MASS CENTERS (FOR ONE SUBJECT) (AFTER HARLESS)

Segment	Distance from Proximal Joint Segment Length = 1.000	Distance from Apex of Head to Sole = 1,000
Head (from apex)	0.361	43.530
Upper trunk	0.497	276.585
Lower trunk	0.518	413.000
R. upper arm	0.427	235.245
L. upper arm	0.432	235.245
R. Forearm	0.417	404.290
L. Forearm	0.402	402.805
R. hand	0.361	608.230
L. hand	0.357	605.245
R. thigh	0.430	571.850
L. thigh	0.569	568.875
R. shank	0.443	841.680
L. shank	0.494	841.680
R. foot (from heel)	0.436	974.955
L. foot	0.436	974.955
Mean for all segments	0.432	

the spheres indicate the location of mass centers (centers of gravity) of the segments.

Modified models of the mass distribution of the human body and mass center location of the segments have been made by several other investigators. It is unfortunate that up to now a unified and universally accepted subdivision of the human body into segments does not exist.

In 1884, C. Meeh investigated the body segment volumes of ten living subjects (8 males and 2 females), ranging in age from 12 to 56 years. In order to approximate the mass of the segments, he determined the specific gravity of the whole body. This was measured during

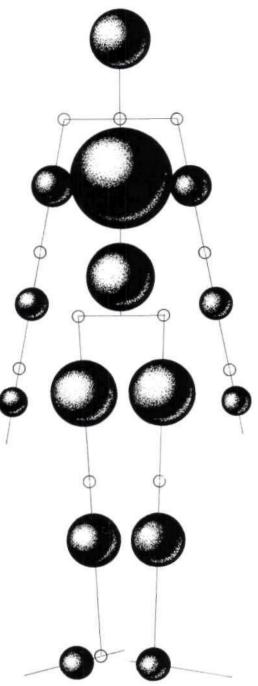


Fig. 3. Body mass distribution (After E. Harless).

quiet respiration and was found to vary between 0.946 and 1.071 and showed no definite variation with age. The segment subdivision used by Meeh is shown in Figure 4 and the

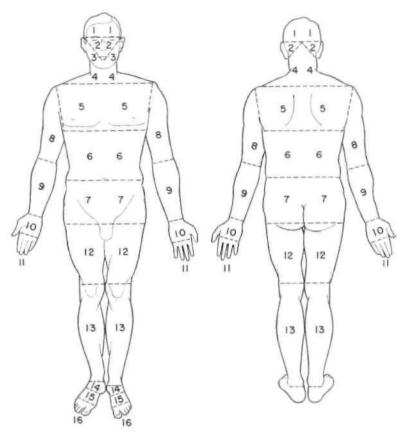


Fig. 4. Body segments (After C. Meeh).

TABLE 5.	RODA	SEGMENT	VOLUME.	TOTAL	RODA	VOLUME	-	1,000	(AFTER	MEEH)	
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Segment	Males (8	Subjects)	Females (2 Subjects)	
Cranium (1)* and upper jaw (2)	71.64	1	57.67	
Lower jaw (3) and neck (4)	38.32		29.83	
Head and neck $(1 + 2 + 3 + 4)$	1	109.96		87.50
Chest (5)	186.10	9.5 1.5 1.5 1	137.76	
Abdomen (6)	137.47		144.68	
Pelvis (7)	182.95		215.83	
Whole trunk $(5 + 6 + 7)$		506.52		498.27
Upper arm (8)	28.04	0.00.00000000000	27.56	NUCLE WARDER TO
Forearm (9)	14.90		13.51	
Palm and thumb (10)	5.20		3.72	1
The four fingers (11)	1.95		2.07	
The whole hand $(10 + 11)$	7.15		5.79	
Both upper extremities	080-0826-9	100.19		93.73
Thigh (12)	81.63		100.42	
Shank (13)	43.56		46.51	
Base of foot (14)	12 77		10.02	
Middle foot (15)∫	13.77		10.92	
The five toes (16)	2.70		2.40	
The whole foot $(14 + 15 + 16)$	16.47		13.32	
Both lower extremities		283.33		320,50
Total body		1,000.00		1,000.00

\* The numbers indicate the segments in Figure 4.

results of the segment volume measurements are presented in Table 5.

C. Spivak, in 1915, in the United States, measured the volumes of various segments and the whole body for 15 males. He found that the value of specific gravity of the whole body ranged from 0.916 to 1.049.

D. Zook, in 1930, made a thorough study of how body segment volume changes with age. In making this study, he used the immersion method for determining segment volumes. These were expressed in per cent of whole body volume. His sample consisted of youngsters between the ages of 5 and 19 years. His immersion technique was unique, but his claim that it permitted the direct determination of the specific gravity of any particular body segment does not seem to have been established. Some of his results are shown in Figures 5 and 6.

In the period from 1952 to 1954, W. Dempster at the University of Michigan made a very thorough study of human body segment measurements. His investigations were based on values obtained on eight cadavers. Besides volumes, he obtained values for mass, density, location of mass center, and mass moments of inertia. The immersion method was used to determine volume. However, these data have limited application since all of Dempster's subjects were over 50 years of age (52-83) and their average weight was only 131.4 lb. The

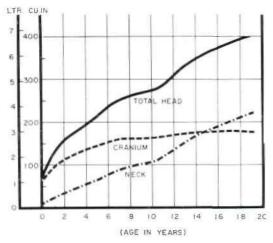


Fig. 5. Mean head volume change with age (After D. Zook)  $% \left( \left( {{{\cal L}_{{\rm{D}}}} \right)_{{\rm{A}}}} \right)$ 

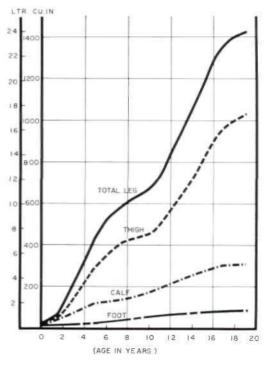


Fig. 6. Mean leg volume change with age (After D. Zook and others).

immersion method was used in Russia by Ivanitzkiy (1956) and Salzgeber (1949).

The immersion technique can be applied for the determination of the total segment volume or any portion thereof in a step-by-step sequence. It can be applied as well on living subjects as on cadavers. In this respect it is a useful technique.

There is some evidence that for most practical purposes the density may be considered constant along the full length of a segment. According to O. Salzgeber (1949), this problem was studied by N. Bernstein in the 1930's before he started his extensive investigations on body segment parameters. By dividing the extremities of a frozen cadaver into zones of 2 cm. height, it was established that the volume centers and mass centers of the extremities were practically coincident. It would seem therefore that the density along the segment was fairly constant for the case studied. Accepting this, it follows that the extremity mass, center of mass, and mass moment of inertia may be determined from the volume data obtained by immersion. However, it should be noted that for the whole body, according to an investigation by Ivanitzkiy (1956), the mass center does not coincide with the volume center, due to the smaller density of the trunk.

# COMPUTATIONAL METHODS

Harless was the first to introduce computational methods as alternatives to the immersion method for determining body volume and mass. He suggested that this would be better for specific trunk segments since no definite marks or anatomical limits need be applied.

He considered the upper part of the trunk down to the iliac crest as the frustum of a right circular cone. The volume (V1) is then determined by the formula:

$$V_1 = \frac{\pi h}{3} (r_1^2 + r_1 r_2 + r_2^2)$$
, where

h is the height of the cone, r<sub>i</sub> is the greater radius, and r<sub>2</sub> is the lesser radius of the cone.

He assumed that the volume of the lower (abdomino-pelvic) part of the trunk (V2) can be approximated as a body between two parallel, nonsimilar elliptical bases with a distance h between them. The volume V2 is determined by the formula:

$$V_2 = \frac{\pi h}{6} [2(ab + a_1b_1) + ab_1 + a_1b] \text{ where }$$

a and b are the half axes of the greater, and  $a_1$  and  $b_1$  are the half axes of the lesser elliptical area, and h is the distance between them.

On the basis of dimensions taken on one subject, using these formulas he arrived at a value for V1 of 21,000 cm cubed and 5,769 cm subed for V2. Using a value of 1.066 gr/cm cubed as the appropriate specific gravity of these parts, the total trunk weight was computed to be 28.515 kg. The actual weight of the trunk was determined (by weighing) to be 29.608 kg. The computed weight thus differed from the actual weight by 1.093 kg, or 3.69 per cent.

Several subsequent investigators used this method subdividing the body into segments of equal height. For increased accuracy these zones should be as small as practically possible —a height of 2 cm is the practical lower limit. The zone markings are measured starting usually from the proximal joint of the body segment. The circumference of the zone is measured and it is assumed that the crosssection is circular. The volume may be computed and on the basis of accepted specific gravity values the mass may be found. From these values one may compute the center of mass and mass moment of inertia.

Amar (1914) in order to compute the mass moment of inertia of various body segments made a number of assumptions. He assumed the trunk to be a cylinder, and that the extremities have the form of a frustum of a cone. The mass moment of inertia for the trunk about a lateral axis through the neck is determined from the formula:

 $I = M/12 (3r^2 + 4h^2)$  where r and h are the radius and height of the cylinder, respectively,

## and for the extremities by the formula:

$$I \approx M\left[\frac{1^2}{9}\left(l+\frac{d}{r+r_1}\right) + \frac{(r+r_1)^2 - 2d^2}{16}\right]$$
 where

- I is the mass moment of inertia about a lateral axis through the proximal joint;
- M is the mass of the segment;
- *l* is the length of the segment or height of the frustum;

r is the radius of the larger cone base;

 $r_1$  is the radius of the smaller cone base;

d is the quantity  $(r - r_1)$ .

Weinbach (1938) proposed a modified zone method based on two assumptions: (1) that any cross-section of a human body segment is elliptical, and (2) that the specific gravity of the human body is uniform in all its segments and equal to 1.000 gr/cm cubed. The area (A) at any cross section is expressed by the equation:

$$A = \pi ab$$
 where a and b are the half axes of the ellipse.

Plotting a graph showing how the equidistant cross-sectional areas change relative to their location from the proximal joint, it is possible to determine the total volume of the segment and hence its mass and location of center of mass. The mass moment of inertia (/) may be obtained by summing the products of the distances from the proximal joint to the zone center squared (*r squred*) and the corresponding zone mass:

 $I = \sum mr^2$ 

Unfortunately both of Weinbach's assumptions are questionable since the cross sections of human body segments are not elliptical and the specific gravities of the different segments are not equal to 1.000 gr/cm cubed nor is density truly uniform in all segments.

Bashkirew (1958) determined the specific gravity of the human body for the Russian population to be 1.044 gr/cm cubed with a standard deviation of  $\pm 0.0131 \text{ gr/cm}$  cubed and the limits from 0.978 minimum to 1.109 maximum. Boyd (1933) determined further that specific gravity generally increases with age. Dempster (1955) showed that Weinbach's method was good for determining the volume of the head, neck, and trunk but not good for other body parts.

It is evident that the determination of body segment parameters, based on the assumption that the segments can be represented by geometric solids, should not be used when great accuracy is desired. This method is useful only when an approximate value is adequate.

Fischer introduced another approximate method of determining human body parameters by computation known as the "coefficient method." According to this procedure, it is assumed that fixed relations exist between body weight, segment length, and the segment parameters which we intend to find. There are three such relationships or ratios expressed as coefficients. For the body segment mass, the coefficient is identified as C1 and represents the ratio of the segment mass to the total body mass. The second coefficient C2 is the ratio of the distance of the mass center from the proximal joint to the total length of the segment. The third coefficient C3 is the ratio of the radius of gyration of the segment about the mediolateral centroidal axis to the total segment length. Thus to determine the mass of a given segment for a new subject, it would be sufficient to multiply his total body mass by coefficient C1 corresponding segment mass. Similarly the location of mass center and radius of gyration can be determined by multiplying the segment length by the coefficients C2 and C3 respectively.

Table 6 compares the values of coefficient *C1* obtained by different investigators.

Table 6 shows that the differences between the coefficients obtained by different investigators for particular segment masses are great. The difference is highest for the trunk and head mass where the coefficients vary from 49.68 to 56.50 per cent of body mass. Next highest difference is in the thigh coefficients from 19.30 to 24.43 per cent of body mass. Since the number of subjects used in the studies, with the exception of that of Bernstein, is small and no anthropological information on body build is given, it is difficult to draw any definite conclusions about the scientific and practical value of these coefficients for body segment mass determination.

As already mentioned, the data obtained by Harless are based on two decapitated male cadavers, and since the blood had been removed some errors are possible. The data of Meeh are based on volume measurements of eight living subjects. The large coefficient for the trunk is influenced by the assumption that all body segments have the same average density, where actually it is less for the trunk.

Braune and Fischer (1889) made a very careful study of several cadavers. Their coefficients are based on data taken on three male cadavers whose weight and height were close to the data for the average German soldier. The relative masses (coefficients) of the segments were expressed in thousandths of the whole body mass. The positions of the mass center and radius of gyration (for determination of the segment mass moments of inertia) were expressed as

TABLE 6. BODY SEGMENT MASS AS PER CENT OF TOTAL MASS (MALES ONLY)

	Investigator						
Segment	Harless	Braune and Fischer	Bern- stein	Demp- ster			
Head, neck, and trunk less limbs	53.42	49.68	52.98	56.50			
Upper arms	6.48	6.72	5.31	5.30			
Forearms	3.62	4.56	3.64	3.10			
Hands	1.68	1.68	1.41	1.20			
Thighs	22.36	23.16	24.43	19.30			
Shanks	8.78	10.54	9.31	9.00			
Feet	3.66	3.66	2.92	2.80			

proportional parts of the segment's total length. Fischer's coefficients have been accepted and used in most subsequent investigations to date.

N. Bernstein and his co-workers (1936) at the Russian All-Union Institute of Experimental Medicine in Moscow carried out an extensive investigation on body segment parameters of living subjects. The study took care of anthropological typology of body build. The results of this investigation were published in a monograph, Determination of Location of the Centers of Gravity and Mass (weight) of the Limbs of the Living Human Body (in Russian). At present the monograph is not available in the United States. Excerpts of this investigation, which cover 76 male and 76 female subjects, 12 to 75 years old, were published by N. Bernstein in 1935 in his chapters on movement in the book, *Physiology of Work* (in Russian), by G. P. Konradi, A. D. Slonim, and V. C. Farfel.

Table 7 shows data for the comparison of segment masses of living male and female subjects as established by Bernstein's investigation. The data are self-explanatory.

# DETERMINATION OF MASS CENTER LOCATION

In the biomechanical analysis of movements it is necessary to know the location of the segment mass center which represents the point of application of the resultant force of gravity acting on the segment. The mass center location of a segment system such as an arm or a leg or the whole body determines the characteristics of the motion.

Table 7. Body Segment as per cent of Total Body Mass (= 100.00)

	Bernstei	Bernstein (Living Subjects)				
Segment	76 Males	76 Females	Average	3 Male Cadavers		
Upper arms	5.31	5.20	5.26	6.72		
Forearms	3.64	3.64	3.64	4.56		
Hands	1.41	1.10	1.26	1.68		
Thighs	24.43	25.78	25.11	23.16		
Shanks	9.31	9.68	9.49	10.54		
Feet	2.92	2.58	2.75	3.66		

TABLE	8.	DISTAN	ICE	OF	MASS	CEN	TER	FROM
THE	PR	OXIMAL	Jo	INT.	Segm	ENT	LEN	GTH
			-	10	00			

1.000

	Investigator						
Segment	Harless	Braune and Fischer	Bernstein	Dempster			
Upper arm	0.485	0.470	0.466	0.436			
Forearm	0.440	0.420	0.412	0.430			
Thigh	0.467	0.440	0.386	0.433			
Shank	0.360	0.420	0.413	0.433			

TABLE 9. DISTANCE OF FOREARM MASS CENTER FROM THE PROXIMAL JOINT. FOREARM LENGTH = 1.000 (AFTER BERNSTEIN)

Age		Male	Female		
Age	Mean M	Range M $\pm \sigma$	Mean M	Range M $\pm \sigma$	
12-15	0.383	0.359-0.407	0.415	0.392-0.441	
16-25	0.419	0.388-0.450	0.417	0.383-0.451	
26-35	0.409	0.383-0.435	0.425	0.388-0.462	
36-45	0.403	0.384-0.422	0.405	0.370-0.440	
46-75	0.428	0.402-0.454	0.411	0.381-0.441	

Table 8 shows the relative location of the mass center for different segments. It is evident that the assumption that mass center of all segments is located 45 per cent from the proximal and 55 per cent from the distal end of the segment is not valid. Since the mass distribution of the body is related to body build it seems that the mass center location also depends on it.

Bernstein claims that he was able to locate the mass centers with an accuracy of  $\pm 1$  mm. Hence the data of Table 9 represent the result of very careful measurements. An analysis of these data shows that there is no definite trend of the coefficients differing with age or sex. The variance of the coefficients is very high and reaches nine per cent as maximum. Thus the use of the same coefficients for subjects with a wide range of body build is highly questionable.

Figures 7 and 8 represent, in modification, Fischer's schemes for the indication of the mass center location of the extremities. The letters of the alphabet indicate the location levels of the mass centers on the human figure. The corresponding cross sections through the segments are shown separately. The letters designate the following:

A-mass center of upper arm

- B-mass center of whole arm
- C-mass center of forearm
- D-mass center of forearm and hand
- E-mass center of hand
- F-mass center of thigh
- G—mass center of whole leg
- H-mass center of shank

I—mass center of shank and foot J—mass center of foot

The location of mass centers with respect to the proximal and distal joints as determined by W. Dempster (1955) is shown in Figure 9.

It is easy to find the equations for the determination of the coordinates of the mass center when the coordinates of the segment's proximal and distal joints are given.

By using Fischer's coefficients for mass center of a particular segment the following

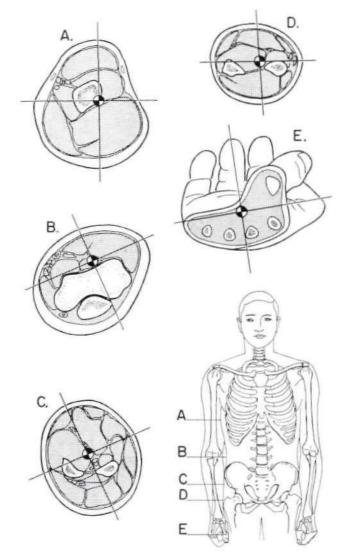


Fig. 7. Location of mass centers of the upper extremity (Redrawn from 0. Fischer).

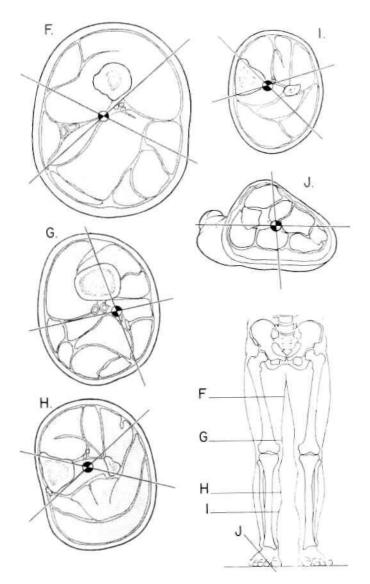


Fig. 8. Location of mass centers of the lower extremity (Redrawn from O. Fischer).

# formulas were developed:

Coordinates of mass center of the: a. forearm:
x = 0.42xd + 0.58xp
1
y = 0.42yd + 0.58yp
where xd, yd are coordinates of the distal
(wrist) joint and xp, yp are coordi-
nates of the proximal (elbow) joint.
nates of the proximal (cloow) joint.
b. upper arm:
x = 0.47xd + 0.53xp
$\mathbf{y} = 0.47 y d + 0.53 x p$

where *xd*, *yd* are coordinates of the elbow joint and *xp*, *yp* are coordinates of the shoulder joint.

c. shank:

x = 0.42dx + 0.58xp
y = 0.42yd + 0.58yp
where xd, yd are coordinates of the ankle joint and xp, yp are coordinates of the knee joint.
d. thigh:

 $\begin{array}{rcl} x &= 0.44xd \,+\, 0.56xp \\ y &= 0.44yd \,+\, 0.56yp \end{array}$ 

where *xd*, *yd* are coordinates of the knee joint and *xp*, *yp* are coordinates of the hip joint.

For the case of three-dimensional recordings of motion, similar equations for z are used. The coordinates of the mass center of trunk (t) are:

$$xt = 0.235 (xfr + xfl) + 0.265 (xbr + xbl),$$
  
with similar equations for the vt and zt coordinates

Here xfr is the coordinate of the right hip and xfl is the coordinate of the left hip, and xbr is the coordinate of the right shoulder and xbl is the coordinate of the left shoulder.

In the same manner the equations for segment systems are developed:

```
a. entire arm:
```

mass center x coordinate given by:

```
xac = 0.130 xgm + 0.148 xm + 0.448 xa + 0.27
xb where
```

*xac*—entire arm mass center *x* coordinate

xgm—mass center of the hand xm—wrist joint xa—elbow joint xb—shoulder joint

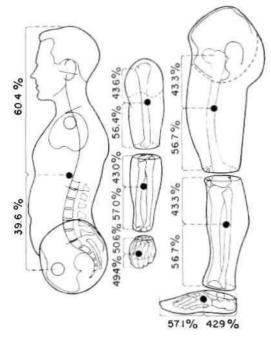


Fig. 9. Location of mass centers of body segments (After W. Dempster).

Similar equations for *y* and *z* coordinates are used:

```
b. entire leg:
mass center x coordinate given by:
xlc = 0.096 xgp+ 0.119 xp + 0.437 xs + 0.348
xf , where
xlc—entire leg mass center x coordinate
xgp—mass center of foot
xp—ankle joint
xs—knee joint
xf—hip joint
```

Similar equations are developed by the y and z coordinates.

By analogy the formulas for coordinates determining the location of the mass center of the entire body in two or three dimensions can be developed.

As regards the coefficient C3, it is known that the mass moment of inertia (*I*) is proportional to the segment's mass and to the square of the segment's radius of gyration (*p*). Fischer found that the radius of gyration for rotation about the axis through the mass center and perpendicular to the longitudinal axis of the segment can be established by multiplying the segment's length (*l*) by the coefficient C3 = 0.3. Hence the mass moment of inertia with respect to the mass center is Ig = mpp = m(0.3l)(0.31) =0.09ml squred.

For the rotation of the segment about its longitudinal axis, Fischer found the coefficient C4 = 0.35, so that the radius of gyration p = 0.35 d, where d is the diameter of the segment.

Since for living subjects the segment rotates about the proximal or distal joint and not the mass center, the mass moment of inertia that we are interested in is greater than Ig by the term *mee*, where *e* is the distance of mass center from the joint. It follows that the mass moment of inertia for segment rotation about the joint is equal to Ij = mpp + mee = m(pp + ee).

#### NEW YORK UNIVERSITY STUDIES

At present the Biomechanics group of the Research Division of the School of Engineering and Science, New York University, is engaged in the determination of volume, mass, center of mass, and mass moment of inertia of living body segments. The methods employed will now be discussed. Some of these techniques are extensions of the methods used by previous researchers; others are procedures introduced by New York University.

# DETERMINATION OF VOLUME

The two methods being investigated by New York University to determine segment volumes are (1) immersion and (2) mono- and stereo-photogrammetry.

# IMMERSION METHOD

The Biomechanics group at New York University uses water displacement as the basis for segment volume determination. However, the procedure differs from that used by previous researchers in that the subject does not submerge his segment into a full tank of water and have the overflow measured. Instead his segment is placed initially in an empty tank which is subsequently filled with water. In this way, the subject is more comfortable during the test, and the segment remains stationary to ensure the proper results.

A variety of tanks for the various segments hand, arm, foot, and leg—has been fabricated. It is desirable that the tank into which the segment is to be immersed be adequate for the extreme limits which may be encountered and yet not so large as to impair the accuracy of the experiments. A typical setup is shown in Figure 10.

The arm is suspended into the lower tank and set in a fixed position for the duration of the test. The tank is then filled to successive predetermined levels at two-centimeter increments from the supply tank of water above. At each level, readings are taken of the height of the water in each tank, using the meter sticks shown. The volume occupied by water between any two levels is found by taking the difference between heights of water levels and applying suitable area factors. Thus to find the volume of the forearm the displacement volume is found for the wrist to elbow levels in the lower tank and between the corresponding levels in the upper tank. The difference between these two volumes is the desired forearm volume.

To find the center of volume obtain volumes in the same manner of consecutive two-centimeter sections of the limb. Assuming the

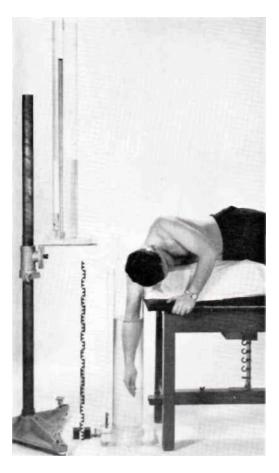


Fig. 10. Determination of the arm volume.

volume center of each section as one centimeter from each face, sum the products of section volume and section moment arm about the desired axis of rotation. The net volume center for the body segment is then this sum divided by the total volume of the segment. In a similar fashion, using the appropriate combination of tanks, we find the volumes of other segments, hand, foot, and leg. The use of an immersion tank to find hand volume is shown in Figure 11. The data on volume and volume centers can also be used along with density as a check against methods of obtaining mass and center of mass.

#### PHOTOGRAMMETRY METHOD

In order to find the volume of an irregularly shaped body part such as the head or face a

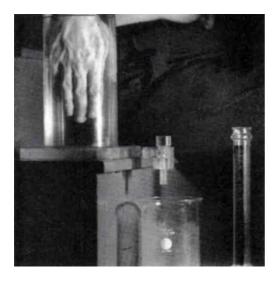


Fig. 11. Determination of the hand volume.

photographic method may be employed. Such a procedure, called photogrammetry, allows not only the volume to be found, but a visual picture of the surface irregularity to be recorded as well. The two types of this technique are mono- and stereophotogrammetry. The principles are the same for each, except that in the latter procedure two cameras are used side by side to give the illusion of depth when the two photographs are juxtaposed. The segment of interest is photographed and the resulting picture is treated as an aerial photograph of terrain upon which contour levels are applied. The portions of the body part between successive contour levels form segments whose volumes can be found by use of a polar planimeter on the photograph as described by Wild (1954). By summing the segmental volumes, the total body segment volume can be found. A controlled experiment by Pierson (1959) using a basketball verified the accuracy of such procedure. Hertzberg, Dupertuis, and а Emanuel (1957) applied the technique to the measurement of the living with great success. The reliability of the photographic technique was proven by Tanner and Weiner (1949). For a more detailed discussion of the photogrammetric method, refer to the paper by Contini, Drillis, and Bluestein (1963).

## METHOD OF REACTION CHANGE

In searching for a method which will determine the segment mass of a living subject with sufficient accuracy, the principle of moments or of the lever has been utilized. The use of this method was suggested by Hebestreit in a letter to Steinhausen (1926). This procedure was later used by Drillis (1959) of New York University. Essentially it consists of the determination of reaction forces of a board while the subject lies at rest on it. The board is supported by a fixed base at one end (A) and a very sensitive weighing scale at the other end (B). The location of the segment center of mass can be found by the methods described elsewhere in this paper. The segment mass is *m*, the mass of the rest of the body is M. The reaction force (measured on the scale) due to the board only should be subtracted from the reaction force due to the subject and board. First the reaction force (S0) is determined when the segment (say the arm) is in the horizontal position and rests alongside the body; second, the reaction force (S) is determined when the segment is flexed vertically to 90 deg. with the horizontal. The distance between the board support points A and B is constant and equal to D. The distance (d) of the segment mass center from the proximal joint is known and the distance b from the proximal joint to support axis A can be measured. From the data it is possible to write the corresponding moment equations about A. The solution of these equations gives the magnitude of the segment's mass as

$$m = \frac{(S - S_0)D}{d}$$

To check the test results, the segment is placed in a middle position, approximately at an angle that is 45 deg. to the horizontal, in which it is held by a special adjustable supporting frame shown at the right in Figure 13.

The magnitude of the segment mass in this case will be determined by the formula:

$$m = \frac{(S_m - S_0)D}{d(l - \cos\varphi)}$$

where  $S_m$  is the measured reaction force for this position, and  $\varphi$  the angle with the horizontal.

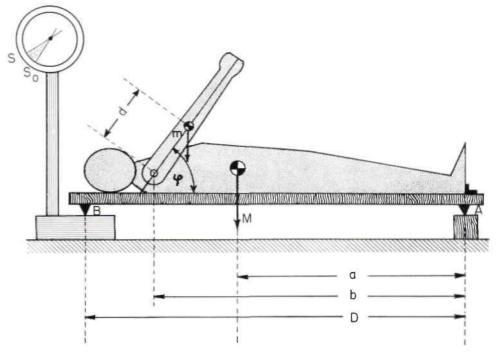


Fig. 12. Determination of the arm mass (reaction board method).

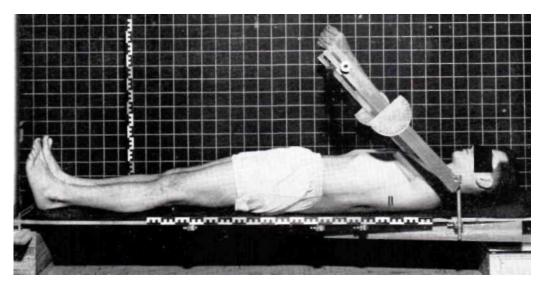


Fig. 13. Reaction board with supporting frame.

By replacing the sensitive scale with an electrical pressure cell or using one force plate, it is also possible to record the changing reaction forces. If the subsequent positions of the whole arm or forearm in flexion are optically fixed as in Stick Diagrams, the corresponding changing reaction forces can be recorded by electrical oscillograph. It is assumed that in flexion the elbow ioint has only one degree of freedom, *i.e.*, it is uniaxial; hence the mass determination of forearm and hand is comparatively simple. The shoulder joint has several degrees of freedom and for each arm position the center of rotation changes its location so that the successive loci describe a path of the instantaneous centers. If the displacement (e) of the instantaneous center in the horizontal direction is known from the Slick Diagram, the magnitude of the segment mass will be

$$m = \frac{(S_m - S_c)D}{d(l - \cos \varphi) + c}$$

where  $S_m$  is the corresponding reaction force for the given position of the segment.

# QUICK RELEASE METHOJD

This technique for the determination of segment moments of inertia is based on Newton's Law for rotation. This law states that the

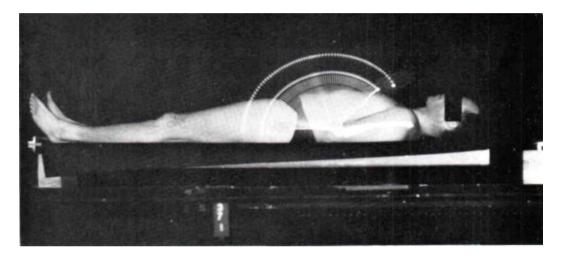


Fig. 14. Stick diagram of forearm flexion.

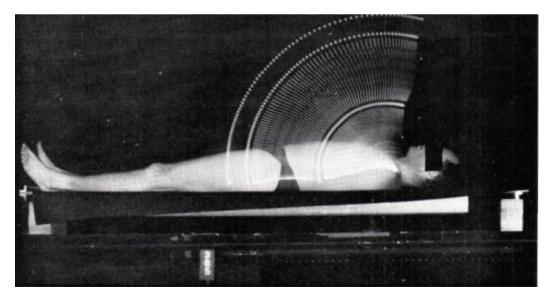


Fig. 15. Stick diagram of arm flexion.

torque acting on a body is proportional to its angular acceleration, the proportionality constant being the mass moment of inertia. Thus if the body segment, say the arm, can be made to move at a known acceleration by a torque which can be evaluated by applying a known force at a given distance, its moment of inertia could be determined. Such a procedure is the basis for the so-called "quick release" method.

To determine the mass moment of inertia of a body segment, the limb is placed so that its proximal joint does not move. At a known distance from the proximal joint at the distal end of the limb, a band with an attached cord or cable is fixed. The subject pulls the cord against a restraint of known force, such as a spring whose force can be found by measuring *its deflection*. The activating torque about the proximal joint is thus proportional to the force and the distance between the joint and the band (moment arm). The acceleration of the limb is produced by sharply cutting the cord or cable. This instantaneous acceleration may be measured by optical or electrical means and the mass moment of inertia about the proximal joint determined.

This technique is illustrated in Figure 16. The subject rotates his forearm about the elbow, thereby pulling against the spring shown at the right through a cord wrapped around a pulley. The mechanism on the platform to the right contains the cutter mechanism with an engagement switch which activates the circuit of the two accelerometers mounted on the subject's forearm. The potentiometer at the base of the spring records the force by measuring the spring's deflection. The accelerometers in tandem give the angular acceleration of the forearm and hand at the instant of cutting. A scale is used to determine the moment arm of the force. This method is further discussed by Drillis (1959).

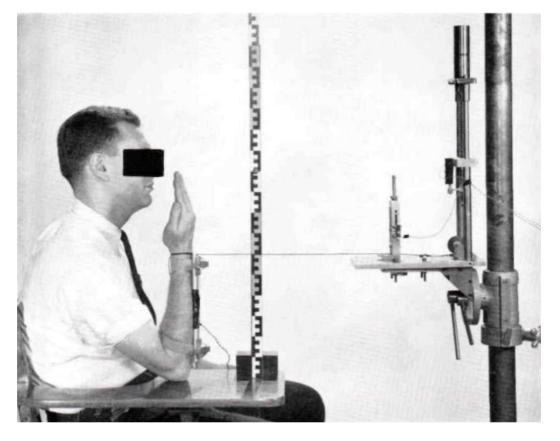


Fig. 16. Quick release method.

# COMPOUND PENDULUM METHOD

This technique for finding both mass moment of inertia of the segment and center of mass may be used in one of two ways: (1) considering the segment as a compound pendulum and oscillating it about the proximal joint, and (2) making a casting of plaster of Paris or dental stone and swinging this casting about a fixed point.

Using the first method, it is necessary to find the moment of inertia, the effective point of suspension of the segment, and the mass center; thus, there are three unknown quantities.

A study by Nubar (1960) showed that these unknowns may be obtained if it is assumed that the restraining moment generated by the individual is negligible. In order to simplify the calculations, any damping moment (resulting from the skin and the ligaments at the joint) is also neglected. The segment is then allowed to oscillate, and its period, or time for a complete cycle, is measured for three cases: (1) body segment alone, (2) segment with a known weight fixed to it at a known point, (3) segment with another known weight fixed at that point. Knowing these three periods and the masses, one can find the effective point of suspension, the center of mass, and the mass moment of inertia from the three equations of motion. If the damping moment at the joint is not negligible, it may be included in the problem as a viscous moment. The above procedure is then extended by the measurement of the decrement in the succeeding oscillations.

In the second procedure, the casting is oscillated about the fixed suspension point. The moment of inertia of the casting is found from the measurement of the period. The mass center can also be determined by oscillating the segment casting consecutively about two suspension points. This method is described in detail by Drillis *el al.* (1963). Since the weight of both the actual segment and cast replica can be found, the measured period can be corrected on the basis of the relative weights to represent the desired parameter (mass center or mass moment of inertia) of the actual segment. The setup for the determination of the period of oscillation is shown in Figure 17.

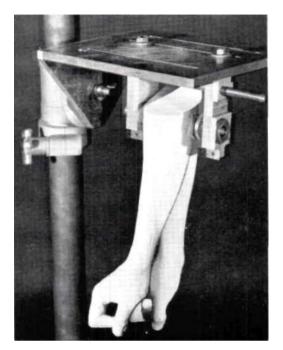


Fig. 17. Compound pendulum method.

The photograph in Figure 17 has been double-exposed to illustrate the plane of oscillation.

#### TORSIONAL PENDULUM METHOD

The torsional pendulum may be used to obtain moments of inertia of body segments and of the entire body. The pendulum is merely a platform upon which the subject is placed. Together they oscillate about a vertical axis. The platform is restrained by a torsion bar fastened to the platform at one end and to the ground at the other. Knowing the physical constants of the pendulum, *i.e.*, of the supporting platform and of the spring or torsion bar, the measurement of the period gives the mass moment of inertia of the whole body. The principle of the torsional pendulum is illustrated schematically in Figure 18.

Figures 19 and 20 describe the setup in use. There are two platforms available: a larger one for studying the supine subject and a smaller one for obtaining data on the erect or crouching subject. In this way, the moments of inertia for both mutually perpendicular axes of the body can be found.

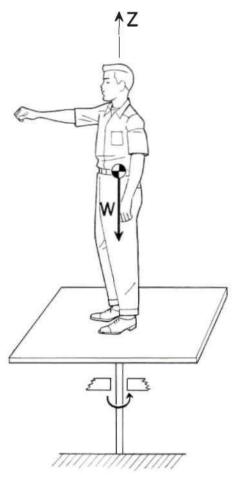


Fig. 18. Torsional pendulum method.

Figure 19 shows a schematic top view of the subject lying supine on the large table. Recording the period of oscillation gives the mass moment of inertia of the body about the sagittal axis for the body position indicated. Figure 20 is a side view of the small table used for the standing and crouching positions. This view shows the torsion bar in the lower center of the picture encased in the supporting structure.

This method can also be used to find mass moments of inertia of body segments. Nubar (1962) describes the necessary procedure and equations. Basically it entails holding the rest of the body in the same position while oscillating the system for two different positions of the segment in question. Knowing the location of the segment in each of these positions, together

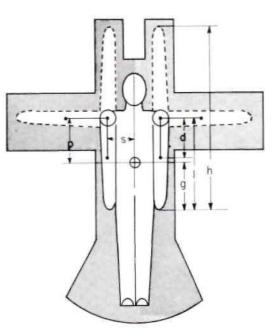


Fig. 19. Body dimensions on torsion table.

with the periods of oscillation of the pendulum, the segment moment of inertia with respect to the mediolateral centroidal axis may be found. This technique is illustrated by the schematic Figure 19 for the case of the arm. The extended position is shown; the period would then be obtained for the case where the arm is placed down at the subject's side.

Both the mass and center of mass of the arm can be determined using the large torsion table. The table and supine subject are rotated for three arm positions—arms at sides, arms outstretched, and arms overhead—and respective total moments of inertia are found from the three periods of oscillation. Assuming that the position of the longitudinal axis of the arm can be defined, *i.e.*, the axis upon which the mass center lies can be clearly positioned, the following equations may be applied:

$$d = \frac{I_2[h(2g + 2l - h) - 4lg] - I_1[p^2 - (h - l - g)^2]}{2I_1(l + g + s - h) + 2I_2(h - 2g) + 2I_3(g - l - s)}$$
  
arm mass = 
$$\frac{I_2 - I_1}{2[p^2 - (l - g)^2 - 2d(g - l - s)]}$$

where *I1*, *I2*, *I3* are the total moments of inertia of table, supports, and subject, found from

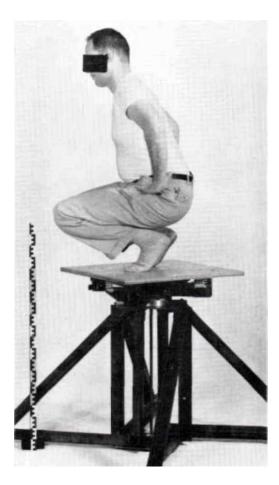


Fig. 20. Mass moment of inertia determination (squatting position).

the periods of oscillation, for the subject with arms at sides, outstretched, and overhead, respectively.

- *h* is the distance from middle fingertip when arms are at the sides to the tip when arms are overhead.
- *l* is the total arm length (fingertip to shoulder joint).
- g is the distance from middle fingertip to the lateral center line of the table when the arms are at the sides.
- *p* is the distance from middle fingertip to the lateral center line when the arms are out-stretched.
- *s* is the distance between the longitudinal center line of the table and the longitudinal axis of the arm when the arms are at the sides.
- *d* is the distance between the mass center of the arm and the shoulder joint.

In this case, the subject is placed so that his total body mass center coincides with the table's fixed point of rotation and there are no initial imbalances. The explanation of the above symbols may be clarified by reference to Figure 19.

# DIFFICULTIES IN OBTAINING PROPER. DATA

In the commonplace technical area, where it has been necessary to evaluate the volume, mass, center of mass, etc., of an inanimate object, this object is usually one of fixed dimensions; that is, there is no involuntary movement of parts. The living human organism, on the other hand, is totally different in that none of its properties is constant for any significant period of time. There are differences in standing erect and in lying down, in inhaling and in exhaling, in closing and in opening the hand. It is necessary, therefore, to develop a procedure of measurement which can contend with these changes, and to evaluate data with particular reference to a specified orientation of the body.

One ever-present problem in dealing with the body is the location of joints. When a segment changes its attitude with respect to adjacent segments (such as the flexion of the elbow), the joint center or center of rotation shifts its position as well. Thus, in obtaining measurements on body segments, it is necessary to specify exactly what the boundaries are. As yet there is no generally accepted method of dividing the body into segments.

When an attempt is made to delineate the boundary between segments for purposes of experimental measurement, one cannot avoid the method of placing a mark on the subject at the joint. This mark will have to serve as the segment boundary throughout the experiment. Unfortunately an error is introduced here when the elasticity of the skin causes the mark to shift as the subject moves. This shift does not correspond to a shift in the actual joint.

In an analysis of a particular body segment involving movement of the segment, such as the quick release, reaction, and torsional pendulum methods which have been described, one must take care to ensure that only the segment moves. Usually this involves both physical and mental preparations on the part of the subject. Finally, the greatest error in obtaining results on body parameters is due to variations in body build. As can be seen from the previous data brought forth, different researchers using identical techniques have gotten quite dissimilar data on the same body segment due to the use of subjects with greatly varying body types.

In an effort to resolve this conflict, the Biomechanics group at New York University is endeavoring to relate their data on body segment parameters to a standard system of body typology.

## ANTHROPOMETRIC STUDIES

In order to develop a means of classifying the subjects according to body build, the method of somatotyping is utilized. Here the body build is designated according to relative amounts of "endomorphy, ectomorphy, and mesomorphy" as described by W. H. Sheldon *et al.* (1940, 1954) in the classic works in the field. In order to determine the subject's somatotype, photographs are taken of three views: front, side, and back. These are illustrated in Figure 21. The Biomechanics group of New York University has obtained the services of an authority in the field, Dr. C. W. Dupertuis, to establish the somatotype of the subjects. The photographs also will be used to obtain certain body measurements.

The aim of the study is to develop relationships between body parameters and body build or important anthropometric dimensions so that a pattern will be established enabling body parameters to be accurately found for all body types.

If sufficient subjects are measured it should be possible to obtain a set of parameter coefficients which take into consideration the effect of the particular body type. When these coefficients are applied to some set of easily measurable body dimensions on any new subject, the appropriate body parameters could easily be determined.

It is planned to prepare tables of these body parameter coefficients (when their validity has been established) for some future edition of *Artificial Limbs*.

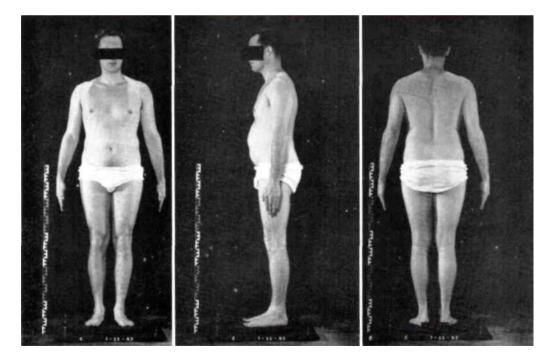


Fig. 21. Photographs for somatotyping.

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